Extension of the Leeson Formula to Phase Noise Calculation in Transistor Oscillators With Complex Tanks

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***Abstract*—For phase noise evaluation in feedback oscillators, the Leeson formula is a useful tool. Nevertheless, a direct application to oscillator circuits with complex feedback tanks can lead to erroneous results because the Leeson formula contains a coefficient, namely, "the loaded quality factor of the circuit," which is not the right coefficient to be used in the expression. It turns out that, in the Leeson formula, this latter coefficient coincides with the loaded Q-factor of the circuit only for specific elemental feedback tanks. We have derived the right coefficient in terms of the energy stored and power dissipated in the circuit. It may be shown that its expression is definitively different from that of the loaded Q-factor of the circuit. We obtain a modified Leeson formula valid for all feedback oscillator circuits. Note that all the electrical models and calculations presented in this paper are valid for FETs as well as for bipolar transistors. A comparison with nonlinear simulations of the phase noise in oscillator circuits with complex feedback tanks demonstrates the validity of the new expression.**

***Index Terms*—Energy stored, Leeson formula, loop gain, oscillation conditions, oscillator Q-factor, oscillators, passive circuit Q-factor, phase noise, phase of loop gain, slope factor.**

I. Introduction

ONE OF THE most important characteristics of an oscillator is its phase noise spectrum. When choosing the architecture, designers must be able to evaluate the phase noise quickly and with sufficient accuracy in order to determine the most convenient architecture for the given application. Then numerical simulations will confirm the first evaluation and will allow the designers to fine tune the circuit.

For a first evaluation of phase noise in feedback oscillators, the Leeson formula is a useful tool [1]. Nevertheless, a direct application of this formula to oscillator circuits with complex feedback tanks can lead to erroneous results because the formula contains coefficients which are not clearly defined. The most troublesome is that one of those coefficients, originally defined by Leeson himself as "the loaded quality factor," is not the right coefficient to be used in the formula. It turns out that the right coefficient coincides with the loaded Q-factor of the circuit only for specific elemental feedback tanks.

However, as a general rule, in complex oscillators designed today and in RF oscillators with distributed elements, a straight application of the Leeson formula can lead to errors of several orders of magnitude (i.e., several tens of decibels) on the phase noise evaluation. This problem has been enlightened during the evaluation of phase noise as a function of the coupling length of the resonator: θ on the oscillator architecture shown in Fig. 1.

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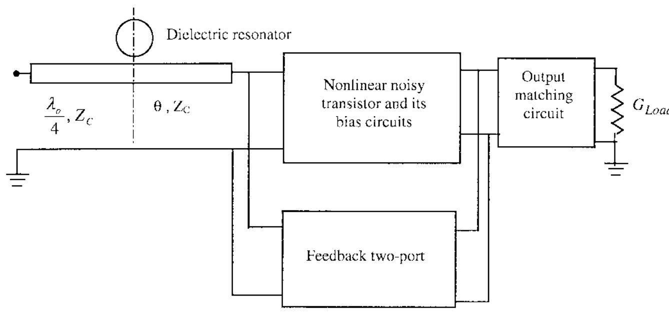

Fig. 1. Nonlinear oscillator circuit under analysis.

If the loaded Q-factor of the circuit is calculated with the conventional rigorous expression (always positive) [2]

$$Q_{\text{loaded}} = \omega_o \frac{\varepsilon}{\overline{P}} \tag{1}$$

where ε is the average energy stored in the circuit and $\overline{P}$ is the average dissipated power, an application of the Leeson formula shows a monotonic decreasing output phase noise as a function of θ.

In contrast to that, measurements which fairly fit nonlinear numerical calculations show a periodic variation of the output phase noise as a function of θ.

In order to elucidate these discrepancies, we have gradually simplified the simulated circuit until obtaining the elemental circuit shown in Fig. 2 whose numerical simulations still reproduce the periodic variation of the output phase noise as function of the coupling length θ, as in the original circuit. The equivalent circuit of the coupled dielectric resonator may be found in [3].

On the other hand this simplified circuit enables us to apply the Leeson formula rigorously after linearization, as shown in Fig. 3.

The discrepancy persisting, we have applied the linear feedback-system formulation to the oscillator circuit of Fig. 3. This calculation shows that in the Leeson formula the loaded Q-factor of the circuit must definitively be replaced by the expression $\omega_o/2\,|d\varphi/d\omega|$, where $|d\varphi/d\omega|$ is the modulus of the group delay (i.e., the phase slope of the loop gain) of the oscillator circuit.

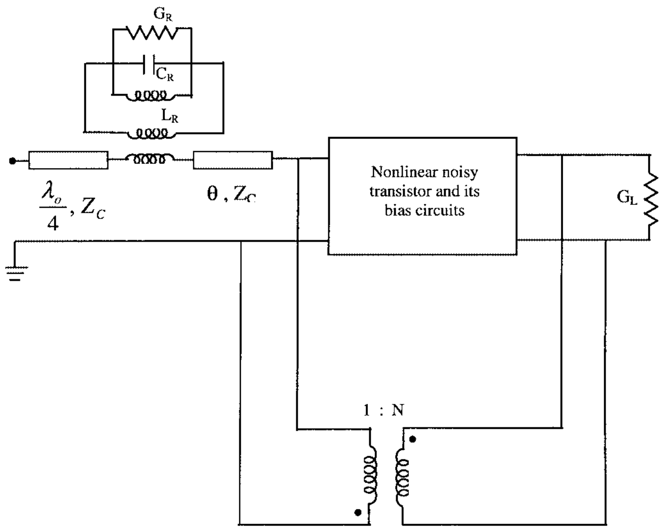

Fig. 2. Simplified nonlinear oscillator circuit.

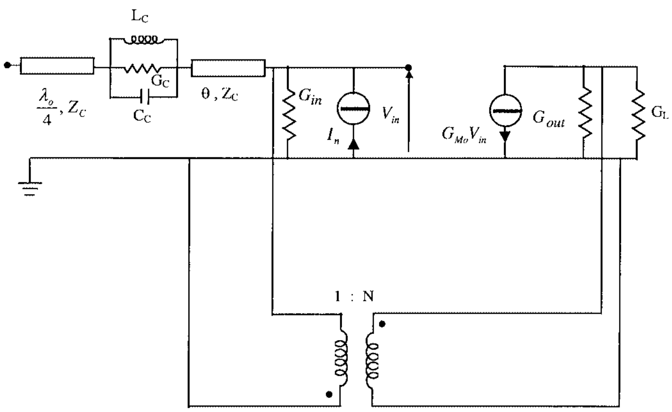

Fig. 3. Simplified linear oscillator circuit.

Although some authors [4] claim that $\omega_o/2\,|d\varphi/d\omega|$ and the Q-factor are two equivalent coefficients, we show in (31) that the general expression of $\omega_o/2\,|d\varphi/d\omega|$, written for a one-port circuit as a function of the energy stored and power dissipated, is definitively different from that of the Q factor given by (32).

It turns out that the expressions of the Q-factor and $\omega_o/2\,|d\varphi/d\omega|$ are identical for some specific elemental circuits. However, as a general rule, these coefficients are very different and using one instead of the other can lead to errors of several tens of decibels in the output phase noise evaluation of an oscillator circuit.

II. Application of the Leeson Formula to the Feedback Oscillator of Fig. 3

In Fig. 3, the open-circuit stub in series with the resonator has a quarter-wave length at the oscillation frequency ω_o, which is also the resonator frequency.

In order to simplify the analytic calculations (without loss of generality), the equivalent conductance G_c of the coupled resonator is taken equal to the characteristic conductance of the coupling line: $G_c = 1/Z_c$; on the other hand, the transmission lines of the circuit are assumed to be lossless.

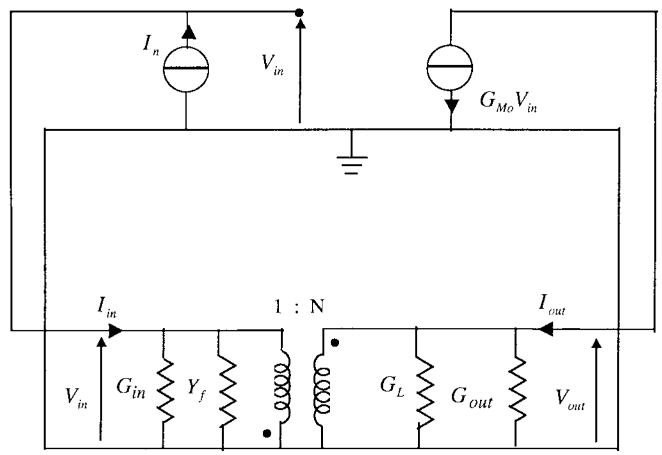

Fig. 4. Equivalent electrical circuit of the feedback oscillator.

The input noise current source I_n represents all the white noise generators of the transistor with their correlations, lumped as one noise source.

$I_n(\omega)$ is such that $\langle I_n(\omega)\, I_n^*(\omega)\rangle$ represents the average power dissipated by the noise source in unit resistance and unit bandwidth, centered at ω [5].

Let us now derive the equations of the circuit.

Fig. 4 shows the equivalent electrical circuit of the feedback oscillator under analysis.

From this figure, we obtain

$$\left.\begin{aligned} V_{\text{in}} &= -\tfrac{V_{\text{out}}}{N} \\ I_n &= -\tfrac{Y_T}{N} V_{\text{out}} + N G_{Mo} V_{\text{in}} \end{aligned}\right\} \tag{2}$$

where $Y_T = Y_f + G_{\text{in}} + N^2\left(G_{\text{out}} + G_{\text{in}}\right)$ and Y_f is the admittance of the whole resonator circuit brought back by the coupling line to the transistor input port.

Around the oscillation frequency, a straightforward calculation gives

$$Y_f = G_c\left\{1 + j\frac{\left[2C_c - \frac{\pi G_c}{2\omega_o}\right]}{G_c} e^{-j\cdot 2\cdot\theta_o}\Delta\omega\right\}. \tag{3}$$

For small offset frequencies $\Delta\omega$ we have

$$Y_f = G_c\left\{1 + j\frac{\left[2C_c - \frac{\pi G_c}{2\omega_o}\right]}{G_c}\cos(2\theta_o)\,\Delta\omega\right\}. \tag{4}$$

The term $2C_c$ is due to the lumped resonator, and $\pi G_c/2\omega_o$ is due to the open circuit stub.

Finally, from (2), we have

$$V_{\text{in}} = \frac{I_n}{Y_T} + \frac{N G_{Mo}}{Y_T} V_{\text{in}}. \tag{5}$$

The closed-loop representation of Fig. 5 can be deduced from (2) and (5).

Let us consider two narrow-band (1 Hz) uncorrelated components of the electrical noise source e_{n-} and e_{n+} respectively located at angular frequencies $\omega_o - \Delta\omega$ and $\omega_o + \Delta\omega$.

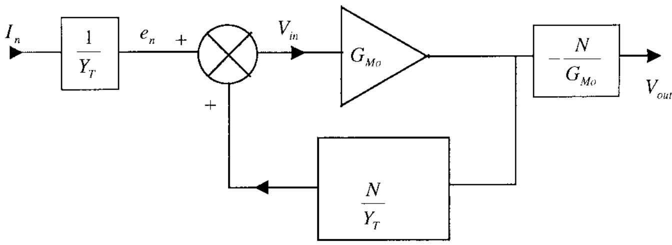

Fig. 5. Closed-loop representation of the oscillator circuit.

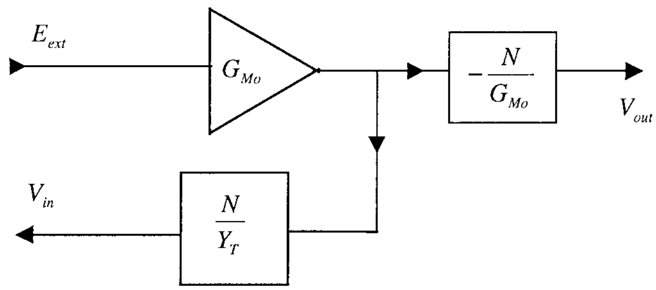

Fig. 6. Open-loop gain calculation by Bode formalism.

A conventional calculation [6], [7] shows that, by addition to a carrier signal of peak value $V_{\text{in}}(\omega_o)$ at the frequency ω_o, these uncorrelated components give rise to a modulated carrier with a phase noise spectral density $S_{\Delta\varphi_{\text{in}}}$, at an offset frequency $\Delta\omega$ from the carrier, given by

$$S_{\Delta\varphi_{\text{in}}} = 2\frac{\left\langle |e_n|^2 \right\rangle}{|V_{\text{in}}(\omega_o)|^2} = 2\frac{\left\langle |I_n|^2 \right\rangle}{|Y_T(\omega_o)|^2 |V_{\text{in}}(\omega_o)|^2} \quad (6)$$

with

$$Y_T(\omega_o) = G_{\text{tot}} = N^2(G_{\text{out}} + G_L) + G_{\text{in}} + G_c. \quad (7)$$

It must be noted that in (6) Y_T is taken at the oscillation frequency ω_o.

This approximation is valid because the transfer function $1/Y_T$ in (5) is not included in the loop: it does not participate in the positive feedback; moreover, the phase noise is calculated for small frequency offset $\Delta\omega$ from the carrier. Then, in (6), Y_T can be written as $Y_T \approx Y_T(\omega_o) = G_{\text{tot}}$.

A. Oscillation Conditions of the Circuit

In order to determine the oscillation conditions, according to Bode [8], the open-loop gain $\tilde{G}$ of the circuit is first calculated.

By setting $I_n = 0$ and opening the feedback loop, we obtain Fig. 6.

The open-loop gain is written as

$$\tilde{G} = \frac{V_{\text{in}}}{E_{\text{ext}}} = \frac{G_{Mo}N}{Y_T} = \frac{G_{Mo}N}{N^2(G_{\text{out}} + G_L) + G_{\text{in}} + Y_f}. \quad (8)$$

At the oscillation frequency, ω_o, $Y_f = G_c$, and we have

$$\tilde{G} = 1 = \frac{G_{Mo}N}{N^2(G_{\text{out}} + G_L) + G_{\text{in}} + G_c}. \quad (9)$$

Note that the oscillation condition (9) is independent of the electrical length θ of the resonator coupling line.

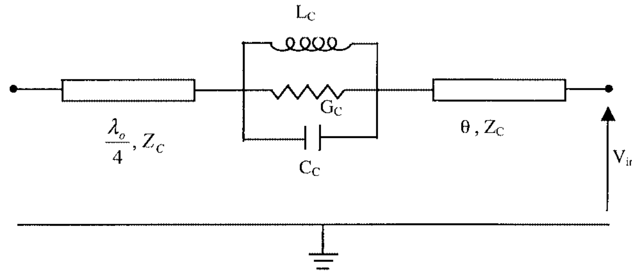

Fig. 7. Calculation of the energy stored in the circuit.

B. Phase Noise Calculation

By applying the Leeson formula, we obtain

$$S_{\Delta\varphi_{\text{out}}} = S_{\Delta\varphi_{\text{in}}}\left[1 + \left(\frac{\omega_o}{2Q_L}\right)^2 \frac{1}{\Delta\omega^2}\right] \quad (10)$$

where $\Delta\omega$ is the frequency offset from the carrier and Q_L is the loaded Q-factor of the circuit.

By definition, the Q_L-factor is rigorously expressed at ω_o by

$$Q_L = \omega_o \frac{\varepsilon \text{ stored in the circuit}}{\overline{P} \text{ total power dissipated}}.$$

From Fig. 3, we can show that the storage elements of the circuit are localized in the feedback tank which includes:

- the open-circuit stub;
- the resonator itself;
- the coupling line of length θ_o

On the other hand, the power is dissipated in the total conductance G_{tot}.

1) Calculation of the Energy Stored in the Circuit: Fig. 7 shows the circuit under consideration. In order to calculate the energy stored in the whole circuit, we successively determine the energy stored in the open-circuit stub, in L_c and C_c, which are the reactive elements of the resonator, and in the transmission line of length θ_o.

2) Energy Stored in the Open-Circuit Stub: The energy stored in a lossless linear two-port circuit is written as [9]

$$\varepsilon = \frac{j}{4}\left[V_1\frac{dI_1^*}{d\omega} + I_1\frac{dV_1^*}{d\omega} + V_2\frac{dI_2^*}{d\omega} + I_2\frac{dV_2^*}{d\omega}\right] \quad (11)$$

where V_1, V_2, I_1, and I_2 are the complex amplitudes of the voltages and currents at the two-port access.

Remembering that the amplitude of the carrier at the input access port is $V_{\text{in}}(\omega_o)$, we have

$$\varepsilon_{\text{stub}}(\omega_o) = \frac{\pi}{8\omega_o}\frac{|V_{\text{in}}(\omega_o)|^2}{Z_c}. \quad (12)$$

3) Energy Stored in the Inductance L_c and Capacitance C_c of the Lumped Resonator: At the resonant frequency, we have

$$\varepsilon_{Lc} = \varepsilon_{Cc} = \frac{1}{4}C_c|V_{\text{in}}(\omega_o)|^2. \quad (13)$$

Therefore,

$$\varepsilon_{\text{tot resonator}} = \frac{1}{2}C_c|V_{\text{in}}(\omega_o)|^2. \quad (14)$$

4) Energy Stored in the Coupling Line θ_o: By applying (11) once again, and after some tedious calculations, we obtain

$$\varepsilon(\theta_o) = \frac{1}{2}\frac{\theta_o}{Z_c\omega_o}\left|V_{\text{in}}(\omega_o)\right|^2 \tag{15}$$

where $\theta_o = \omega_o L/v$ is the electrical length of the coupling line at ω_o.

Finally, the energy stored in the whole circuit is written as

$$\varepsilon_{\text{tot}} = \frac{1}{2}\left|V_{\text{in}}(\omega_o)\right|^2\left[C_c + \frac{1}{Z_c\omega_o}\left(\frac{\pi}{4}+\theta_o\right)\right]. \tag{16}$$

On the other hand, the power dissipated in the circuit is written as

$$\overline{P}(\omega_o) = \frac{1}{2}\left|V_{\text{in}}(\omega_o)\right|^2 G_{\text{tot}}. \tag{17}$$

Finally, the loaded Q is written as

$$Q_L = \omega_o\frac{\left[C_c + \frac{1}{Z_c\omega_o}\left(\frac{\pi}{4}+\theta_o\right)\right]}{G_{\text{tot}}}. \tag{18}$$

A strict application of the Leeson formula yields

$$S_{\Delta\varphi_{\text{out}}} = S_{\Delta\varphi_{\text{in}}}\left\{1+\left[\frac{G_{\text{tot}}}{2C_c + \frac{2}{Z_c\omega_o}\left(\frac{\pi}{4}+\theta_o\right)}\right]^2\frac{1}{\Delta\omega^2}\right\}. \tag{19}$$

According to this expression, the output phase noise should decrease monotonically with θ_o.

Nevertheless, numerical simulations which fit the measurements fairly well show that the output phase noise varies periodically with θ_o.

In order to elucidate these discrepancies, the phase noise of the oscillator circuit is now calculated analytically using the linear feedback-system formalism.

C. Phase-Noise Derivation Using the Linear Feedback-System Formalism

In the context of a linear determination of the phase noise using the feedback-system formalism, one can now represent the oscillator around the oscillation frequency ω_o by the normalized closed-loop representation of Fig. 8.

In order to simplify the notation of the following calculations, we write $\sqrt{S_{\Delta\varphi_{\text{in}}}} = \Delta\varphi_{\text{in}}$. Note that only $S_{\Delta\varphi_{\text{in}}}$ has a physical significance; $\Delta\varphi_{\text{in}}$ is not useful in itself.

$H(\Delta\omega)$ is deduced from (4), (8), and (9).

The normalized transfer function is written as

$$H(\Delta\omega) = \frac{1}{1+j\frac{d\varphi}{d\omega}\Delta\omega} = \frac{1}{1+j\left[\frac{2\cdot C_c - \frac{\pi G_c}{2\omega_o}}{G_{\text{tot}}}\right]\cos(2\theta_o)\,\Delta\omega} \tag{20}$$

where $-d\varphi/d\omega$ is the group delay of the oscillator loop.

The expression of $H(\Delta\omega)$ given by (20) is all the more valid as, when using the Leeson formula, calculations are linear and focused on the phase/frequency relationship around the oscillation frequency.

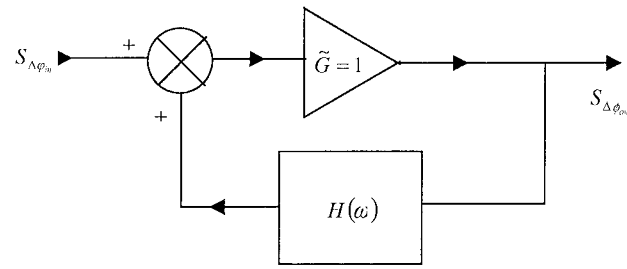

Fig. 8. Closed-loop representation of the feedback oscillator for phase-noise calculation by the linear feedback-system formalism.

From Fig. 8, we obtain successively

$$\Delta\varphi_{\text{in}} = \Delta\varphi_{\text{out}}\left(1-\frac{1}{1+j\frac{d\varphi}{d\omega}\Delta\omega}\right)$$

$$\Delta\varphi_{\text{out}} = \Delta\varphi_{\text{in}}\left(1+j\frac{1}{\frac{d\varphi}{d\omega}\Delta\omega}\right)$$

and finally

$$S_{\Delta\varphi_{\text{out}}} = S_{\Delta\varphi_{\text{in}}}\left(1+\frac{1}{\left[\frac{d\varphi}{d\omega}\right]^2\Delta\omega^2}\right). \tag{21}$$

Equation (21) is the correct formula for the evaluation of output phase noise in any kind of feedback oscillator circuit.

$d\varphi/d\omega$ can be easily determined in a feedback oscillator with a complex feedback tank by opening the oscillator loop and by calculating the phase slope of the loop gain at the oscillation frequency.

For verification's sake of the general validity of (21), and in addition to the example detailed in this paper, we have applied the modified formula to several types of oscillator circuits with complex two-port feedback tanks including lumped elements and transmission lines. The good agreement obtained by comparison with numerical simulations confirms the validity of (21).

Although (10) and (21) are similar for some elemental circuits, their values can differ by several orders of magnitude for complex or distributed circuits.

Applying (21) to our example in Fig. 3 yields

$$S_{\Delta\varphi_{\text{out}}} = S_{\Delta\varphi_{\text{in}}}\left(1+\left[\frac{G_{\text{tot}}}{2C_c - \frac{\pi G_c}{2\omega_o}}\right]^2\frac{1}{\cos^2(2\theta_o)\,\Delta\omega^2}\right). \tag{22}$$

This expression can be compared with the result given by (19). It must be noted that, if the resonator is directly coupled without any line or stub, then (19) and (22) coincide.

III. Numerical Simulations

Fig. 1 shows the circuit simulated with a fully nonlinear model of HEMT including five nonlinearities. The output phase noise is calculated in the load conductance G_L.

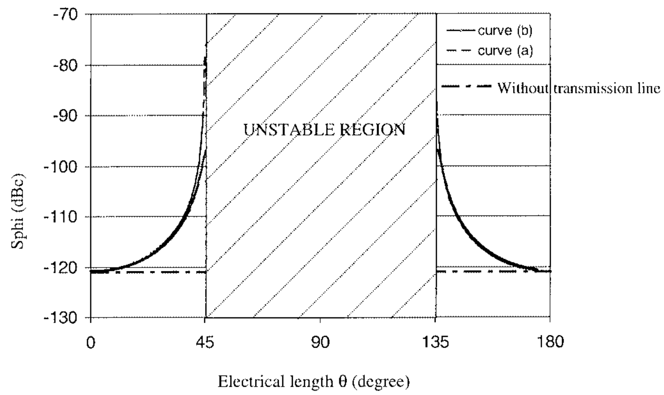

Fig. 9. Simulated phase noise $S_{\Delta\phi_{out}}$ at 100 kHz from the carrier as a function of the electrical length of the resonator transmission line for the white noise source.

From nonlinear stability considerations, which are outside of the scope of this paper, we must have

$$\cos(2\theta_o) \geq 0. \tag{23}$$

Therefore, for stable operation, the physical (i.e., positive) electrical length θ_o must be taken within the following ranges:

$$\left.\begin{array}{c} 0 < \theta_o > \frac{\pi}{4} \\ \text{or} \\ \frac{3\pi}{4} < \theta_o > \pi \end{array}\right\} \text{modulo } \pi. \tag{24}$$

Simulations have been performed with a nonlinear frequency-domain simulator [11]. This simulator is based on the Harmonic Balance principle and related techniques. The simulation of phase noise is based on the conversion and correlation matrices concepts. The validity of the principles used in this simulator has been successfully checked with many benchmarks by comparing its results with that of a time-domain Monte Carlo simulator. The main characteristics of this nonlinear simulator may be found in [12].

In our circuit (Fig. 1), two noise sources have been successively simulated:

- a white noise source;
- a $1/f$ noise source.

As expected, a phase noise roll-off, respectively, of -20 db/dec and -30 db/dec versus frequency offset from carrier is obtained.

Besides, for a fixed frequency offset, both noise sources give similar curves as a function of the electrical length θ_o.

Fig. 9 shows the results obtained at 100 kHz from the carrier with the white noise source: curve (a) is the representation of (22) and curve (b) is the simulation result.

The small difference between the two curves is due to the nonlinear effects in the transistor. The figure speaks for itself and confirms the result of (21).

Application of the Leeson formula gives a phase noise of -121 dbc practically constant as a function of θ_o. Note that the unloaded Q-factor of the resonator used was $Q_o = 3000$.

IV. General Expression of the Group Delay as a Function of the Energy Stored and Power Dissipated in a One-Port Linear Circuit

For comparison purposes with the passive circuit Q-factor, the general expression of the group delay of a linear one-port circuit is now derived as a function of the energy stored and power dissipated.

Conventional electromagnetic theory demonstrates that the admittance of a linear one-port circuit may be written as [10]

$$Y(\omega) = \frac{2j\omega(\varepsilon_e - \varepsilon_M) + \overline{P}}{\frac{1}{2}|E|^2} \tag{25}$$

where

ε_e average electric energy stored in the circuit;
ε_M average magnetic energy stored in the circuit;
$\overline{P}$ average power dissipated;
E amplitude of the driving generator.

At the resonant frequency of the circuit ω_o, the admittance becomes purely real and

$$\varepsilon_e(\omega_o) = \varepsilon_M(\omega_o). \tag{26}$$

Around ω_o, Y can be written as

$$Y(\Delta\omega) = |Y|\, e^{j(d\varphi_Y/d\omega)\cdot\Delta\omega}. \tag{27}$$

$Y(\omega)$ is the transfer function of the one-port circuit and is defined as

$$H(\omega) = Y(\omega) = \frac{I\,(\text{response})}{E\,(\text{driving signal})}. \tag{28}$$

The modulus of the group delay is

$$|\tau| = \left|\frac{d\angle H}{d\omega}\right|. \tag{29}$$

From (25)–(27), we obtain at resonance

$$\left.\frac{d\varphi_Y}{d\omega}\right|_{\omega_o} = \frac{2\omega_o \frac{d}{d\omega}(\varepsilon_e - \varepsilon_M)}{\overline{P}(\omega_o)} \tag{30}$$

and

$$\frac{\omega_o}{2}\left|\frac{d\varphi_Y}{d\omega}\right|\Bigg|_{\omega_o} = \frac{\omega_o^2}{\overline{P}(\omega_o)}\left|\frac{d}{d\omega}(\varepsilon_e - \varepsilon_M)\right|. \tag{31}$$

On the other hand, the Q-factor of the circuit is

$$Q = \frac{\omega_o(\varepsilon_e + \varepsilon_M)}{\overline{P}(\omega_o)}. \tag{32}$$

Equation (31) equals (32) only for some elemental circuits in which

$$\left|\frac{d(\varepsilon_e - \varepsilon_M)}{d\omega}\right|_{\omega_o} = \frac{\varepsilon_e + \varepsilon_M}{\omega_o}.$$

However, in general, we have

$$Q \neq \frac{\omega_o}{2}\left|\frac{d\varphi_Y}{d\omega}\right|.$$

An application of (31) and (32) to some examples of one-port circuits is given in the Appendix.

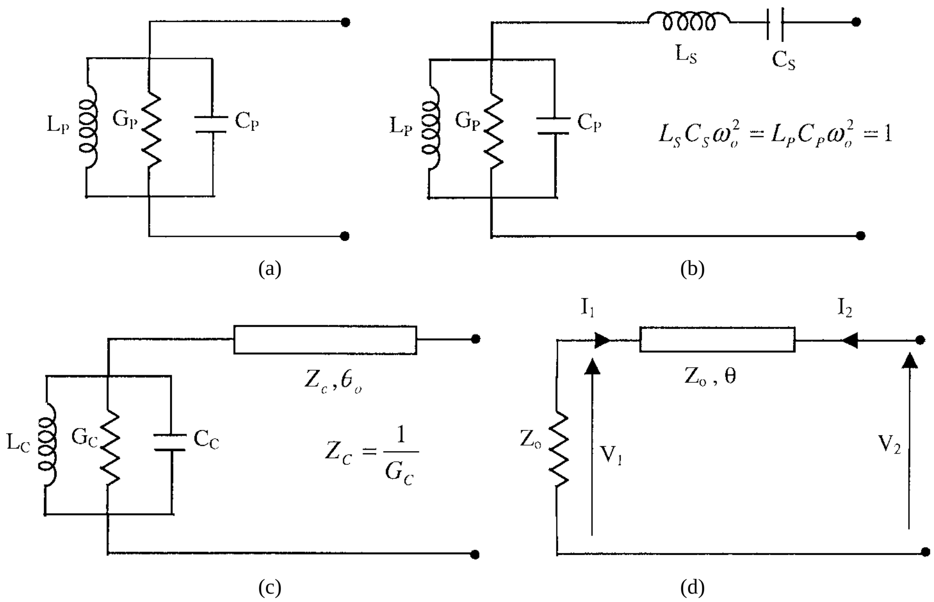

Fig. 10. (a) Parallel resonant circuit. (b) Parallel resonant circuit with a lossless series resonant circuit. (c) Parallel resonant circuit with a transmission line. (d) One-port matched line circuit.

V. Conclusion

An extension of the Leeson formula to the evaluation of phase noise in feedback oscillators with complex feedback tanks is proposed.

The agreement obtained by a comparison between the new formula and a rigorous nonlinear numerical calculation shows that this new analytic expression can be a useful and accurate tool for the design of solid-state RF and microwave oscillators.

On the other hand, a general expression of the group delay of a linear passive one-port circuit has been derived in terms of energy stored and power dissipated. An application of this expression to some common circuits is reported in the Appendix and compared to the Q-factor of the corresponding circuits.

Appendix
Group Delay and Q-Factor of Some One-Port Circuits

Let us look at one-port passive circuits driven by an ac generator. $d\varphi/d\omega$ and Q are successively calculated for the one-port circuit under consideration.

1) Parallel Resonant Circuit [Fig. 10(a)]: Fig. 10(a) shows the conventional parallel resonant circuit.

From (30) and (31) of the main text, we obtain at ω_o

$$\frac{d\varphi_Y}{d\omega} = \frac{1}{G_p} 2C_p$$
$$Q = \frac{\omega_o}{2 \cdot G_p} 2C_p = \frac{\omega_o C_p}{G_p}.$$

For this elemental circuit, $\omega_o/2\,|d\varphi_Y/d\omega| = Q$.

2) Parallel Resonant Circuit in Series with a Lossless Series Resonant Circuit [Fig. 10(b)]: For this circuit, at ω_o, we have

$$\frac{d\varphi_Y}{d\omega} = \frac{1}{G_p}\left[2C_p - 2G_p^2 L_S\right].$$

The susceptance slope of the series-tuned circuit compensates for the susceptance slope of the parallel-tuned circuit.

On the other hand, the Q-factor writes

$$Q = \frac{\omega_o\left[C_p + G_p^2 L_S\right]}{G_p}.$$

In this circuit, $\omega_o/2\,|d\varphi_Y/d\omega|$ and the Q-factor are two different coefficients.

3) Transmission Line of Electrical Length θ_o Loaded by a Parallel Resonant Circuit [Fig. 10(c)]: In this example, where $G_c = 1/Z_c$, at the resonant frequency of the resonator we obtain

$$\frac{d\varphi_Y}{d\omega} = \frac{2C_c}{G_c}\cos(2\theta_o)$$

and

$$\frac{\omega_o}{2}\left|\frac{d\varphi_Y}{d\omega}\right| = \frac{\omega_o C_c}{G_c}\left|\cos(2\theta_o)\right|.$$

On the other hand, the Q-factor writes:

$$Q = \frac{\omega_o}{G_c}\left[C_c + \frac{G_c\theta_o}{\omega_o}\right].$$

Once again, $\omega_o/2\,|d\varphi_Y/d\omega|$ and the Q-factor are very different.

4) The Matched Line [Fig. 10(d)]: A lossless transmission line of characteristic impedance Z_o and electrical length θ, loaded by Z_o, is finally investigated.

From (11), the energy stored in the line at ω_o is written as

$$\varepsilon_{\text{stored}} = \frac{1}{2}\frac{|V_1|^2}{Z_o}\left.\frac{d\theta}{d\omega}\right|_{\omega_o} = \frac{1}{2}\frac{|V_1|^2}{Z_o}\frac{l}{v}.$$

On the other hand,

$$\overline{P} = \frac{1}{2}\frac{|V_1|^2}{Z_o}.$$

Let us write $\theta_o = \omega_o l/v$, then we have

$$Q_{\text{loaded line}} = \theta_o$$

while

$$Z_{\mathrm{port}(1)} = Z_o$$

and

$$\frac{d}{d\omega}(\varepsilon_M - \varepsilon_e) = 0.$$

Then

$$\frac{d\varphi_z}{d\omega} = 0.$$

For this instructive example, we have

$$\frac{\omega_o}{2}\left|\frac{d\varphi_z}{d\omega}\right| = 0$$

and

$$Q_{\mathrm{loaded\,line}} = \theta_o.$$

In order to demonstrate definitively that the Q-factor of a circuit is not the correct coefficient to be used in the Leeson formula, we have replaced the resonator and its coupling line of Fig. 2 by a matched line and performed a numerical simulation.

By varying the length θ (from $\theta = 0$ up to 1000 rad) we have found, as expected, that:

- the output phase noise was extremely high: it was only dependent on the reactive elements of the transistor and its matching circuits;
- the output phase noise was independent of θ.

In conclusion, (31) and (32) are two different coefficients characterizing a one-port linear circuit. Their evaluation generally gives different results which are equal only for some specific elemental circuits.

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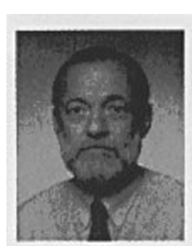

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